

### Some Well Known and Essential Series

(1) Geometric Series:  $\sum_{n=1}^{\infty} ar^{n-1} = a + ar + \dots + ar^{n-1} + \dots$

if  $(a \neq 0; a, r: \text{constants})$

(i) Convergent (Converges to  $\frac{a}{1-r}$ ), if

$|r| < 1$  and (ii) Divergent, if  $|r| > 1$ .

(2) The p-Series:  $\sum_{n=1}^{\infty} \frac{1}{n^p} = 1 + \frac{1}{2^p} + \frac{1}{3^p} + \dots + \frac{1}{n^p} + \dots$

if  $(p \text{ constant})$

(i) Convergent, if  $p > 1$  and (ii) Divergent, if  $p \leq 1$

(3) Harmonic Series (The p-Series with  $p=1$ )

$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} + \dots$

if Divergent Series.

30	31					1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
S	M	T	W	T	F	S

## The Series

$$\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p} = \frac{1}{2(\log 2)^p} + \frac{1}{3(\log 3)^p} + \dots \rightarrow \infty$$

is Convergent or divergent  
according as  $p > 1$  or  $p \leq 1$

Example! —

(i)  $1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots \rightarrow \infty$  ( $p = \frac{1}{2} < 1$ ) is divergent

(ii)  $1 + \frac{1}{2} + \frac{1}{3} + \dots \rightarrow \infty$  ( $p = 1$ , i.e. Harmonic series)  
is divergent.

(iii)  $1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots \rightarrow \infty$  ( $p = 2 > 1$ ) is Convergent

(iv)  $1 + \frac{1}{2^7} + \frac{1}{3^7} + \dots \rightarrow \infty$  ( $p = 7$ ) is Convergent

(v)  $1 + \frac{1}{2^{5/2}} + \frac{1}{3^{5/2}} + \dots \rightarrow \infty$  ( $p = \frac{5}{2} > 1$ ) is  
Convergent.